Enhanced Turing Machines

Lecture 28
Sections 10.1 - 10.2

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Outline

Increasing the Power of a Turing Machine

- It is hard to believe that something as simple as a Turing machine could be powerful enough to solve complicated problems.
- We can imagine a number of improvements.
  - A Stay option
  - Multiple tapes
  - One-way infinite tape
  - Two-dimensional tape (*n*-dimensional tape)
  - Addressable memory
  - Nondeterminism
  - etc.

Outline

A Stay Option

- Rather more left or right on every transition, we could allow the Turing machine to stay at its current tape position.

A Stay Option

Theorem

Any Turing machine with a Stay option is equivalent to some standard Turing machine.

Outline

One-way Infinite Tape

- Would a Turing machine with a one-way infinite tape be more powerful than a standard Turing machine?

One-way Infinite Tape

Theorem

Any Turing machine with a one-way infinite tape is equivalent to a standard Turing machine.

One-way Infinite Tape

- We can use a one-tape machine to simulate the two-way infinite tape.
- Establish a tape position that marks the left “edge” of the one-way tape.
- Symbols that would be to the right of that mark will occupy the odd positions.
- Symbols that would be to the left of that mark will occupy the even positions.
- Modity the transitions accordingly.

Outline

1. Variants of Turing Machines
   - A Stay Option
   - One-Way Infinite Tape
   - Multiple Tapes
   - Examples

2. Assignment

Multiple Tapes

- Would a Turing machine with k tapes, $k > 1$, be more powerful than a standard Turing machine?
- Each tape could be processed independently of the others.
- In other words, each transition would read each tape, write to each tape, and move left or right independently on each tape.

Multiple Tapes

Theorem

Any language that is accepted by a multitape Turing machine is also accepted by a standard Turing machine.

Multiple Tapes

Proof (sketch):

- Let the tape contents be:

$$
\begin{array}{ll}
\text{Tape 1:} & x_{11}x_{12}x_{13}\ldots x_{1n_1} \\
\text{Tape 2:} & x_{21}x_{22}x_{23}\ldots x_{2n_2} \\
\vdots & \vdots \\
\text{Tape } k\text{:} & x_{k1}x_{k2}x_{k3}\ldots x_{kn_k}
\end{array}
$$

- Then we would write on a single tape

$$\#x_{11}x_{12}\ldots x_{1n_1}\#x_{21}x_{22}\ldots x_{2n_2}\#\ldots\#x_{k1}x_{k2}\ldots x_{kn_k}\#.$$

□

Multiple Tapes

Proof (sketch):

- To show the current location on each tape, put a special mark _ on one of that tape's symbols:

$$\#x_{11}\underline{x}_{12}\ldots x_{1n_1}\#\underline{x}_{21}x_{22}\ldots x_{2n_2}\#\ldots\#x_{k1}\underline{x}_{k2}\ldots x_{kn_k}\#$$

- Begin with

$$\#\underline{x}_{11}x_{12}\ldots x_{1n_1}\#\sqcup\#\ldots\#\sqcup\#$$

□

Multiple Tapes

Proof (sketch):

- The Turing machine scans the tape, locating the current symbol on each “tape.”
- It then makes the appropriate transition.
  - Write a symbol in each of the current positions.
  - Move the location of the current symbol one space left or right for each tape.
- Of course, the devil is in the details.

□

Outline

1. Variants of Turing Machines
   - A Stay Option
   - One-Way Infinite Tape
   - Multiple Tapes
   - Examples

2. Assignment

Binary Addition

Example (Binary Addition)

- Binary addition is much simpler if we have a three-tape machine.
- Write x on tape 1 and write y on tape 2.
- Write the sum on tape 3.
- For simplicity, assume fixed-length integers.

Outline

Assignment

Homework

- Use JFLAP to do the following:
  - Design a 3-tape machine that will do subtraction of fixed-length integers. Allow the results to "wrap around." That is, if $y > x$, then the result of $x - y$ will be $2^n + (x - y)$.
  - Design a 3-tape machine that will accept the language

$$\{\mathbf{a}^n\mathbf{b}^n\mathbf{c}^n \mid n \geq 0\}.$$